

(with  $D_{ls}$  and  $D_{cs}$  similarly defined), we obtain

$$\text{Maximize } \sum_{s \in \{l, m, h\}} (-2x_{ls} - 5.2x_{cs} - 4x_{fs} + 60s_{ds} + 40s_{ls} + 10s_{cs})p_s \quad (P.1)$$

subject to

$$\begin{aligned} -x_{ls} + 8y_{ds} + 6y_{ls} + y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ -x_{cs} + 2y_{ds} + 1.5y_{ls} + 0.5y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ -x_{fs} + 4y_{ds} + 2y_{ls} + 1.5y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ s_{ds} &\leq D_{ds}, & s \in \{l, m, h\}, \\ s_{ds} - y_{ds} &\leq 0, & s \in \{l, m, h\}, \\ s_{ls} &\leq D_{ls}, & s \in \{l, m, h\}, \\ s_{ls} - y_{ls} &\leq 0, & s \in \{l, m, h\}, \\ s_{cs} &\leq D_{cs}, & s \in \{l, m, h\}, \\ s_{cs} - y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ x_{ls}, x_{fs}, x_{cs}, y_{ds}, y_{ls}, y_{cs}, s_{ds}, s_{ls}, s_{cs} &\geq 0, & s \in \{l, m, h\}. \end{aligned}$$

As indicated, (P.1) is separable by scenario. We can consider each demand scenario separately, and we can obtain scenario-specific solutions independently. Only in calculating the objective value do we combine them. At the other extreme, we determine both acquisition and production before we know the demand (2 in Figure 2) (Figure 4).

Once made, the decisions about acquisition and production are fed into the demand uncertainty. Only the sales levels respond to the acquisition and production levels and the manner in which the demand uncertainty is resolved. Any LP model of this problem must capture the fact that the initial decisions must be weighed against all possible demand scenarios. To accomplish this, we use three separate sets of the sell variables, and only one set of the acquisition and production variables. As before, we work from (P.0) to develop our model. To connect Figure 4 and the LP model, we use a **bold** font to identify decisions made

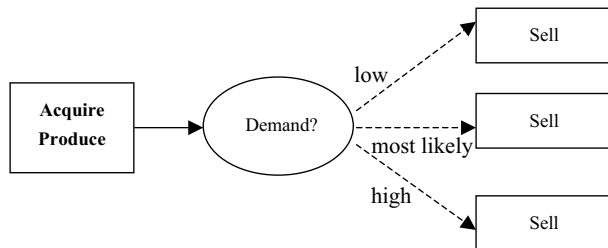


Figure 4: If demand is known after acquisition and production are determined, it will only affect the amount of product that is sold.

before demand is known.

$$\text{Maximize } -2x_l - 5.2x_c - 4x_f + \sum_{s \in \{l, m, h\}} (60s_{ds} + 40s_{ls} + 10s_{cs})p_s \quad (P.2)$$

subject to

$$\begin{aligned} -x_l + 8y_d + 6y_t + y_c &\leq 0 \\ -x_c + 2y_d + 1.5y_t + 0.5y_c &\leq 0 \\ -x_c + 4y_d + 2y_t + 1.5y_c &\leq 0 \\ s_{ds} &\leq D_{ds}, & s \in \{l, m, h\}, \\ s_{ds} &\leq 0, & s \in \{l, m, h\}, \\ -y_d & & s_{ls} \leq D_{ls}, & s \in \{l, m, h\}, \\ & & s_{ls} \leq 0, & s \in \{l, m, h\}, \\ y_t & & s_{cs} \leq D_{cs}, & s \in \{l, m, h\}, \\ & & s_{cs} \leq 0, & s \in \{l, m, h\}, \\ -y_c & & & \\ x_l, x_f, x_c, y_d, y_t, y_c, s_{ds}, s_{ls}, s_{cs} &\geq 0, & s \in \{l, m, h\}. \end{aligned}$$

In contrast to (P.1), (P.2) is not separable by scenario. Acquisition and production, represented by  $x$  and  $y$ , are determined before demand is known and are held constant across all scenarios. The second set of constraints models the manner in which sales depend on the combination of production and demand. The lack of separability arises because of the interaction of the two types of variables in these constraints.

Finally, in the remaining case (3 in Figure 2), we determine acquisition before we know the demand and production and sales afterward (Figure 5)

As we work from (P.0) to develop an LP model for this problem, we have a single set of acquisition variables, and three sets of production and sales variables:

$$\text{Maximize } -2x_l - 5.2x_c - 4x_f + \sum_{s \in \{l, m, h\}} (60s_{ds} + 40s_{ls} + 10s_{cs})p_s \quad (P.3)$$

subject to

$$\begin{aligned} -x_l + 8y_{ds} + 6y_{ls} + y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ -x_c + 2y_{ds} + 1.5y_{ls} + 0.5y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ -x_f + 4y_{ds} + 2y_{ls} + 1.5y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ s_{ds} &\leq D_{ds}, & s \in \{l, m, h\}, \\ s_{ds} - y_{ds} &\leq 0, & s \in \{l, m, h\}, \\ s_{ls} &\leq D_{ls}, & s \in \{l, m, h\}, \\ s_{ls} - y_{ls} &\leq 0, & s \in \{l, m, h\}, \\ s_{cs} &\leq D_{cs}, & s \in \{l, m, h\}, \\ s_{cs} - y_{cs} &\leq 0, & s \in \{l, m, h\}, \\ x_l, x_f, x_c, y_{ds}, y_{ls}, y_{cs}, s_{ds}, s_{ls}, s_{cs} &\geq 0, & s \in \{l, m, h\}. \end{aligned}$$

Similar to (P.2), (P.3) lacks separability. In general, separability does not occur when the LP model

$$\begin{aligned}
 & b_{12E}^{n1} + b_{12E}^{n2} < 3/2, \quad b_{12E}^{n1} < 1, \quad b_{12L}^{n1} = 0, \quad b_{12L}^{n2} = 0; \\
 & b_{13E}^{n1} + b_{13E}^{n2} < 3/2, \quad b_{13E}^{n1} < 2/3, \quad b_{13L}^{n1} = 0, \quad b_{13L}^{n2} = 0; \\
 & b_{14E}^{n1} + b_{14E}^{n2} < 3/2, \quad b_{14E}^{n1} < 1/3, \quad b_{14L}^{n1} = 0, \quad b_{14L}^{n2} = 0; \\
 & b_{15E}^{n1} + b_{15L}^{n1} < 5/6, \quad b_{15E}^{n1} < 1/2, \quad b_{15E}^{n2} = 0, \quad b_{15L}^{n2} = 0; \\
 & b_{16E}^{n1} + b_{16E}^{n2} + b_{16L}^{n1} < 5/6, \quad b_{16E}^{n1} + b_{16E}^{n2} < 1/2, \\
 & \quad b_{16E}^{n1} + b_{16L}^{n1} < 1/2, \quad b_{16E}^{n1} < 1/6, \quad b_{16L}^{n2} = 0; \\
 & b_{17E}^{n1} + b_{17L}^{n1} + b_{17L}^{n2} < 1, \quad b_{17E}^{n1} + b_{17L}^{n1} < 1/2, \quad b_{17E}^{n2} = 0; \\
 & b_{18E}^{n1} + b_{18E}^{n2} + b_{18L}^{n1} < 1/2, \quad b_{18E}^{n1} + b_{18L}^{n1} + b_{18L}^{n2} < 2/3, \\
 & \quad b_{18E}^{n1} + b_{18L}^{n1} < 1/6; \\
 & b_{19E}^{n1} + b_{19L}^{n1} + b_{19L}^{n2} < 1, \quad b_{19E}^{n1} + b_{19L}^{n1} < 1/6, \quad b_{19E}^{n2} = 0; \\
 & b_{20E}^{n1} + b_{20L}^{n1} < 4/3, \quad b_{20E}^{n1} < 1, \quad b_{20E}^{n2} = 0, \quad b_{20L}^{n2} = 0; \\
 & b_{21E}^{n1} + b_{21E}^{n2} + b_{21L}^{n1} < 4/3, \quad b_{21E}^{n1} + b_{21E}^{n2} < 1, \\
 & \quad b_{21E}^{n1} + b_{21L}^{n1} < 1, \quad b_{21E}^{n1} < 2/3, \quad b_{21L}^{n2} = 0; \\
 & b_{22E}^{n1} + b_{22E}^{n2} + b_{22L}^{n1} < 4/3, \quad b_{22E}^{n1} + b_{22E}^{n2} < 1, \\
 & \quad b_{22E}^{n1} + b_{22L}^{n1} < 2/3, \quad b_{22E}^{n1} < 1/3, \quad b_{22L}^{n2} = 0; \\
 & b_{23E}^{n1} + b_{23L}^{n1} + b_{23L}^{n2} < 3/2, \quad b_{23E}^{n1} + b_{23L}^{n1} < 1, \quad b_{23E}^{n2} = 0; \\
 & b_{24E}^{n1} + b_{24E}^{n2} + b_{24L}^{n1} < 1, \quad b_{24E}^{n1} + b_{24L}^{n1} + b_{24L}^{n2} < 7/6, \\
 & \quad b_{24E}^{n1} + b_{24L}^{n1} < 2/3; \\
 & b_{25E}^{n1} + b_{25E}^{n2} + b_{25L}^{n1} < 1, \quad b_{25E}^{n1} + b_{25L}^{n1} + b_{25L}^{n2} < 5/6, \\
 & \quad b_{25E}^{n1} + b_{25L}^{n1} < 1/3; \\
 & b_{26E}^{n1} + b_{26L}^{n1} + b_{26L}^{n2} < 3/2, \quad b_{26E}^{n1} + b_{26L}^{n1} < 2/3, \quad b_{26E}^{n2} = 0; \\
 & b_{27E}^{n1} + b_{27E}^{n2} + b_{27L}^{n1} < 2/3, \quad b_{27E}^{n1} + b_{27L}^{n1} + b_{27L}^{n2} < 7/6, \\
 & \quad b_{27E}^{n1} + b_{27L}^{n1} < 1/3; \\
 & b_{28E}^{n1} + b_{28L}^{n1} + b_{28L}^{n2} < 3/2, \quad b_{28E}^{n1} + b_{28L}^{n1} < 1/3, \quad b_{28E}^{n2} = 0.
 \end{aligned}$$

Combining these with Equations (A5)–(A9), we obtain

$$\begin{aligned}
 & 2x_4^n/3 + 2x_6^n/3 + 3x_8^n/4 \\
 & \leq y_1^n/2 + y_2^n/6 + y_3^n/2 + y_4^n/6 + y_5^n \\
 & \quad + 2y_6^n/3 + y_7^n/3 + y_8^n + 2y_9^n/3 + y_{10}^n/3 \\
 & \quad + 4y_{11}^n/3 + y_{12}^n + 2y_{13}^n/3 + y_{14}^n/3 + y_{15}^n/2 \\
 & \quad + y_{16}^n/6 + y_{17}^n/2 + y_{18}^n/6 + y_{19}^n/6 + y_{20}^n \\
 & \quad + 2y_{21}^n/3 + y_{22}^n/3 + y_{23}^n + 2y_{24}^n/3 + y_{25}^n/3 \\
 & \quad + 2y_{26}^n/3 + y_{27}^n/3 + y_{28}^n/3 + O(1), \quad (A10)
 \end{aligned}$$

$$\begin{aligned}
 & 2x_4^n/3 + 2x_5^n/3 + 2x_6^n/3 + 2x_7^n/3 + 3x_8^n/4 + 2x_{10}^n/3 \\
 & \leq y_1^n/2 + y_2^n/2 + y_3^n/2 + y_4^n/2 + y_5^n + y_6^n + y_7^n + y_8^n + y_9^n \\
 & \quad + y_{10}^n + 4y_{11}^n/3 + 3y_{12}^n/2 + 3y_{13}^n/2 + 3y_{14}^n/2 + y_{15}^n/2 \\
 & \quad + y_{16}^n/2 + y_{17}^n/2 + y_{18}^n/2 + y_{19}^n/6 + y_{20}^n + y_{21}^n + y_{22}^n + y_{23}^n \\
 & \quad + y_{24}^n + y_{25}^n + 2y_{26}^n/3 + 2y_{27}^n/3 + y_{28}^n/3 + O(1), \quad (A11)
 \end{aligned}$$

$$\begin{aligned}
 & 2x_4^n/3 + 2x_6^n/3 + 3x_8^n/4 + 3x_9^n/4 \\
 & \leq y_1^n/2 + y_2^n/6 + y_3^n/2 + y_4^n/6 + y_5^n + 2y_6^n/3 + y_7^n/3 + y_8^n \\
 & \quad + 2y_9^n/3 + y_{10}^n/3 + 4y_{11}^n/3 + y_{12}^n + 2y_{13}^n/3 + y_{14}^n/3 \\
 & \quad + 5y_{15}^n/6 + y_{16}^n/2 + y_{17}^n/2 + y_{18}^n/6 + y_{19}^n/6 + 4y_{20}^n/3 \\
 & \quad + y_{21}^n + 2y_{22}^n/3 + y_{23}^n + 2y_{24}^n/3 + y_{25}^n/3 \\
 & \quad + 2y_{26}^n/3 + y_{27}^n/3 + y_{28}^n/3 + O(1), \quad (A12)
 \end{aligned}$$

$$\begin{aligned}
 & 2x_4^n/3 + 2x_5^n/3 + 2x_6^n/3 + 2x_7^n/3 + 3x_8^n/4 + 3x_9^n/4 + 2x_{10}^n/3 \\
 & \leq y_1^n/2 + y_2^n/2 + y_3^n/2 + y_4^n/2 + y_5^n + y_6^n + y_7^n + y_8^n + y_9^n \\
 & \quad + y_{10}^n + 4y_{11}^n/3 + 3y_{12}^n/2 + 3y_{13}^n/2 + 3y_{14}^n/2 + 5y_{15}^n/6 \\
 & \quad + 5y_{16}^n/6 + y_{17}^n/2 + y_{18}^n/2 + y_{19}^n/6 + 4y_{20}^n/3 + 4y_{21}^n/3 \\
 & \quad + 4y_{22}^n/3 + y_{23}^n + y_{24}^n + y_{25}^n + 2y_{26}^n/3 + 2y_{27}^n/3 \\
 & \quad + y_{28}^n/3 + O(1), \quad (A13)
 \end{aligned}$$

$$\begin{aligned}
 & 2x_4^n/3 + 2x_6^n/3 + 3x_8^n/4 + 3x_9^n/4 + 2x_{11}^n/3 \\
 & \leq y_1^n/2 + y_2^n/6 + y_3^n/2 + y_4^n/6 + y_5^n + 2y_6^n/3 + y_7^n/3 \\
 & \quad + y_8^n + 2y_9^n/3 + y_{10}^n/3 + 4y_{11}^n/3 + y_{12}^n + 2y_{13}^n/3 + y_{14}^n/3 \\
 & \quad + 5y_{15}^n/6 + y_{16}^n/2 + y_{17}^n + 2y_{18}^n/3 + y_{19}^n + 4y_{20}^n/3 + y_{21}^n \\
 & \quad + 2y_{22}^n/3 + 3y_{23}^n/2 + 7y_{24}^n/6 + 5y_{25}^n/6 + 3y_{26}^n/2 \\
 & \quad + 7y_{27}^n/6 + 3y_{28}^n/2 + O(1). \quad (A14)
 \end{aligned}$$

By the definition in the beginning, we also have

$$\sum_{i=1}^{28} y_i^n = n + O(1). \quad (A15)$$

When the 1-pieces are not present, we further have

$$x_2^n + x_3^n + x_6^n + x_7^n + y_3^n + y_4^n + y_8^n + y_9^n + y_{10}^n = 0. \quad (A16)$$

Hence,  $(\sum_{i=1}^{11} x_i^n) / (\sum_{i=1}^{28} y_i^n)$  is upper-bounded by  $\max\{\sum_{i=1}^{11} x_i^n \mid x_i^n \geq 0, i = 1, \dots, 11, y_i^n \geq 0, i = 1, \dots, 28, (A1)-(A4), (A10)-(A16)\} / n$ . The limit of this upper bound is another linear program of the apparent form. By the simplex method, we find its solution value to be  $\frac{35}{18}$ .  $\square$

We believe  $R_{MXF}^\infty \leq 35/18$  even when there are 1-pieces in the list. However, without being enforced by (A16), the linear program gives the result of  $22/9 \simeq 2.4444$  and a solution vector indicating that the valid constraints we have come up with so far have not captured the delicate timing of the appearances of the 1-pieces in both the MXF and OPT packings. Therefore, it calls for more subtle arguments than our existing ones to prove the result for the case with 1-pieces. Also, letting the underlying CBP algorithm be FF instead of NF is important because only the former allows the lower bound of  $3/4$  relevant to  $a_{8E}^{n1}$  and  $a_{9L}^{n1}$ . When FF is replaced by NF, we can only achieve a lower bound of  $2$  using similar arguments.

PROOF OF THEOREM (6). The idea here is to have  $20K$  pieces around  $\frac{1}{6}$ ,  $20K$  pieces around  $\frac{1}{3}$ ,  $20K$  pieces around

PROOF OF COROLLARY 1. Substituting into (12) the value of  $\varphi_{NE}^*$  from (10) and  $\alpha = \alpha_1 = \alpha_2$  yields

$$C_{NE}^* = \frac{t_p [2 - \alpha - G(\varphi_{NE}^*)]^2 G(\varphi_{NE}^*)}{4[\alpha + G(\varphi_{NE}^*)]} [1 + 2\varphi_{NE}^* T(\varphi_{NE}^*)] - f.$$

When  $\alpha \rightarrow 0$ , the above payoff  $C_{NE}^* \rightarrow (t_p/4)[2 - G(\varphi_{NE}^*)]^2 [1 + 2\varphi_{NE}^* T(\varphi_{NE}^*)]$ , which is strictly decreasing in  $\varphi_{NE}^*$  if the function

$$\varphi T(\varphi) = \frac{\varphi G'(\varphi)}{G}(\varphi) - \frac{1}{2}$$

is nonincreasing in  $\varphi$ . Hence, if the elasticity of  $G(\cdot)$  is nonincreasing and  $\alpha$  is sufficiently small, the payoff of each station declines when producers advertise more intensely.  $\square$

PROOF OF LEMMA 2.

(i) The expressions for  $\varphi_{ED}^*$  and  $p_{ED}^*$  follow from (18) and (19). Using these expressions, the profit of each station is found by evaluating the symmetric agreement payoff  $C_{ED}^* = a_i^j \varphi_i^j |_{\varphi_{ED}^*, p_{ED}^*}$ , where  $a_i^j \varphi_i^j$  is the negotiation outcome given by (17).

(ii) The expressions for  $\varphi_{ES}^*$ ,  $p_{ES}^*$ , and  $p_{2ES}^*$  follow from (21)–(23) upon the substitution  $X^1 = X^2 = 1/2$  and  $\varphi_2^2 = \varphi_2^1 = \varphi_{ES}^*$ . Using these three expressions, the profit of each station is found by evaluating the symmetric agreement payoff  $C_{ES}^* = a_2^j \varphi_2^j |_{\varphi_{ES}^*, p_{ES}^*, p_{2ES}^*}$ , where  $a_2^j \varphi_2^j$  is the negotiation outcome given by (20).  $\square$

PROOF OF PROPOSITION 2.

(i) The inequality follows because  $T(\cdot)$  is strictly decreasing, by (A2).

(ii) Averaging the prices of the two producers under regime  $ES$  yields

$$\begin{aligned} (\bar{p}_{ES}^* - c) &= \frac{t_p(2\alpha - \alpha^2 - \alpha G_{ES}^* + G_{ES}^*)}{\alpha(\alpha + G_{ES}^*)} \\ &< \frac{t_p(2\alpha - \alpha^2 - \alpha G_{ED}^* + G_{ED}^*)}{\alpha(\alpha + G_{ED}^*)} = (p_{ED}^* - c), \end{aligned}$$

provided  $\alpha > 0$ . The inequality is implied by the fact that  $G_{ED}^* < G_{ES}^*$  from part (i). A comparison with the product price of the  $NE$  regime that is expressed by (11) yields the remaining conclusion because  $G_{ES}^* = G_{NE}^*$  from part (i).

(iii) Evaluating the profits of each station in the limit when  $\alpha \rightarrow 0$  yields

$$\lim_{\alpha \rightarrow 0} C_{NE}^* = \frac{t_p(2 - G_{NE}^*)^2}{4} \left[ 1 + \frac{\varphi_{NE}^*}{t_s} \right] - f,$$

$$\lim_{\alpha \rightarrow 0} C_{ED}^* = \lim_{\alpha \rightarrow 0} \frac{t_p G_{ED}^*}{4\alpha} - f,$$

and

$$\lim_{\alpha \rightarrow 0} C_{ES}^* = \lim_{\alpha \rightarrow 0} \frac{2t_p G_{ES}^*}{9\alpha} \left( 1 + \frac{\varphi_{ES}^*}{t_s} \right) - f.$$

(Note that for existence of an interior equilibrium in the  $ES$  regime, it is required that  $\alpha \neq 0$ . However, we can evaluate the limit expression above since an equilibrium exists for all  $\alpha > 0$  in the neighborhood around 0.)

Because the limit of  $C_{NE}^*$  approaches a finite number whereas  $C_{ED}^*$  and  $C_{ES}^*$  approach infinity, it is clear that  $C_{NE}^* < C_{ED}^*$  and  $C_{NE}^* < C_{ES}^*$  when  $\alpha$  is very small.

As well, it follows from the above that

$$\lim_{\alpha \rightarrow 0} \frac{C_{ED}^* + f}{C_{ES}^* + f} = \frac{9G_{ED}^*}{8G_{ES}^*} \left( 1 + \frac{\varphi_{ES}^*}{t_s} \right)^{-1}.$$

When  $t_s$  approaches infinity,

$$\begin{aligned} \lim_{t_s \rightarrow \infty} G_{ES}^* &= \lim_{t_s \rightarrow \infty} G \left( T^{-1} \left( \frac{1}{2t_s} \right) \right) \\ &= \lim_{t_s \rightarrow \infty} G \left( T^{-1} \left( \left( \frac{4 - \alpha}{2 - \alpha} \right) \frac{1}{2t_s} \right) \right) = \lim_{t_s \rightarrow \infty} G_{ED}^*, \end{aligned}$$

and

$$\lim_{t_s \rightarrow \infty} \left[ 1 + \frac{\varphi_{ES}^*}{t_s} \right] = 1,$$

because  $\varphi_{ES}^*$  is finite. As a result,

$$\lim_{\alpha \rightarrow 0} \frac{C_{ED}^* + f}{C_{ES}^* + f} = \frac{9}{8}.$$

By continuity, therefore,  $C_{ED}^* > C_{ES}^*$  for very small values of  $\alpha$  and very large values of  $t_s$ . From the expressions derived for  $C_{NE}^*$ ,  $C_{ES}^*$ , and  $C_{ED}^*$  it follows that

$$\frac{C_{NE}^* + f}{C_{ES}^* + f} = \frac{9[2 - (\alpha + G_{NE}^*)]^2}{2\alpha(\alpha + G_{NE}^*)} \left[ \frac{2 - \alpha}{\alpha} + \frac{[2 - (\alpha + G_{NE}^*)]}{2(\alpha + G_{NE}^*)} \right]^{-2}$$

and

$$\begin{aligned} \frac{C_{NE}^* + f}{C_{ED}^* + f} &= \frac{2[2 - (\alpha + G_{NE}^*)]^2 G_{NE}^* (\alpha + G_{ED}^*)}{G_{ED}^* (\alpha + G_{NE}^*) (2\alpha - \alpha^2 + G_{ED}^* - \alpha G_{ED}^*)} \\ &\quad \cdot \left[ 1 + \frac{\varphi_{NE}^*}{t_s} \right] \left[ \frac{2 - \alpha}{\alpha} - \frac{\varphi_{ED}^*}{t_s} \right]^{-1}. \end{aligned}$$

Evaluating the above ratios for very large values of  $\alpha \rightarrow 1$  and very small values of  $t_s \rightarrow 0$  (implying that  $G_{NE}^* = G_{ED}^* = 0$ ) yields

$$\lim_{\alpha \rightarrow 1} \frac{C_{NE}^* + f}{C_{ES}^* + f} = 2,$$

$$\lim_{t_s \rightarrow 0} \frac{C_{NE}^* + f}{C_{ED}^* + f} = 2.$$

By continuity, therefore, when  $\alpha$  is sufficiently big and  $t_s$  is sufficiently small,  $C_{NE}^* > C_{ES}^*$  and  $C_{NE}^* > C_{ED}^*$ .  $\square$

PROOF OF LEMMA 3. Without any loss of generality, let the asymmetric regime be characterized by Advertiser 2 advertising on both stations and Advertiser 1 advertising only on Station 1, as illustrated in Figure 2. The symmetric relationship of all agents in this regime implies that three distinct bargaining outcomes must be derived. We first specify agreement and disagreement shares for all three negotiations.

In the negotiations between Advertiser 2 and the two stations, the following market share expressions apply in case of agreement:

$$D_2^1 = \left[ [1 - \alpha - G(\varphi_1^1)] + (\alpha + G(\varphi_1^1)) \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] [\alpha + G(\varphi_2^1)],$$

$$D_2^2 = \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] [\alpha + G(\varphi_2^2)],$$

$$X^1 = \frac{1}{2} + (\varphi_2^2 - \varphi_2^1 - \varphi_1^1)/(2t_s), \quad X^2 = 1 - X^1.$$

In case of disagreement between Advertiser 2 and Station 1,

$$\begin{aligned}\tilde{D}_2^1 &= \left[ [1 - \alpha - G(\varphi_1^1)] + (\alpha + G(\varphi_1^1)) \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \alpha, \\ \tilde{D}_2^2 &= D_2^2, \\ \tilde{X}^1 &= \frac{1}{2} + (\varphi_2^2 - \varphi_1^1) / (2t_s), \quad \tilde{X}^2 = 1 - \tilde{X}^1.\end{aligned}$$

In case of disagreement between Advertiser 2 and Station 2,

$$\begin{aligned}\tilde{D}_2^1 &= D_2^1, \\ \tilde{D}_2^2 &= \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \alpha, \\ \tilde{X}^1 &= \frac{1}{2} - (\varphi_1^1 + \varphi_2^1) / (2t_s), \quad \tilde{X}^2 = 1 - \tilde{X}^1.\end{aligned}$$

In the negotiations between Producer 1 and Station 1, the following market shares apply:

$$\begin{aligned}D_1^1 &= \left[ [1 - \alpha - G(\varphi_2^1)] + (\alpha + G(\varphi_2^1)) \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right] [\alpha + G(\varphi_1^1)], \\ D_1^2 &= \left[ [1 - \alpha - G(\varphi_2^2)] + [\alpha + G(\varphi_2^2)] \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right] \alpha, \\ X^1 &= \frac{1}{2} + (\varphi_2^2 - \varphi_2^1 - \varphi_1^1) / (2t_s), \quad X^2 = 1 - X^1.\end{aligned}$$

In case of disagreement between Advertiser 1 and Station 1,

$$\begin{aligned}\tilde{D}_1^1 &= \left[ [1 - \alpha - G(\varphi_2^1)] + (\alpha + G(\varphi_2^1)) \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right] \alpha, \\ \tilde{D}_1^2 &= D_1^2, \\ \tilde{X}^1 &= \frac{1}{2} + (\varphi_2^2 - \varphi_2^1) / (2t_s), \quad \tilde{X}^2 = 1 - \tilde{X}^1.\end{aligned}$$

Using the above expressions in (4) and (5), the Nash bargaining solution yields the negotiated rates  $a_i^j$ , as follows:

$$\begin{aligned}a_1^1 \varphi_1^1 &= \frac{(p_1 - c)}{2} \left\{ \left( \frac{1}{2} + \frac{\varphi_2^2 - \varphi_2^1 - \varphi_1^1}{2t_s} \right) \right. \\ &\quad \cdot \left[ (1 - \alpha - G(\varphi_2^1)) + (\alpha + G(\varphi_2^1)) \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right] \\ &\quad \cdot G(\varphi_1^1) + \frac{\alpha \varphi_1^1}{2t_s} [G(\varphi_2^1) - G(\varphi_2^2)] \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \left. \right\}. \\ a_2^1 \varphi_2^1 &= \frac{(p_2 - c)}{2} \left\{ \left( \frac{1}{2} + \frac{\varphi_2^2 - \varphi_2^1 - \varphi_1^1}{2t_s} \right) \right. \\ &\quad \cdot \left[ (1 - \alpha - G(\varphi_1^1)) + (\alpha + G(\varphi_1^1)) \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \\ &\quad \cdot G(\varphi_2^1) + \frac{\varphi_2^1}{2t_s} \left( \alpha G(\varphi_1^1) \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right. \\ &\quad \left. \left. + G(\varphi_2^2) \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \right) \right\}.\end{aligned}$$

$$\begin{aligned}a_2^2 \varphi_2^2 &= \frac{(p_2 - c)}{2} \left\{ \left( \frac{1}{2} + \frac{\varphi_2^1 + \varphi_1^1 - \varphi_2^1}{2t_s} \right) \right. \\ &\quad \cdot \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \\ &\quad \cdot G(\varphi_2^2) + \frac{\varphi_2^2}{2t_s} \left( -G(\varphi_1^1) [\alpha + G(\varphi_2^1)] \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right. \\ &\quad \left. \left. + G(\varphi_2^1) \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \right) \right\}.\end{aligned}$$

Substituting the above rates into the first-order conditions with respect to  $\varphi_i^j$  and  $p_i$  (conditions (8) and (9)) yields the following five expressions to determine  $\varphi_2^1, \varphi_2^2, \varphi_1^1, p_1$ , and  $p_2$ .

$$\begin{aligned}\frac{\partial F_1}{\partial \varphi_1^1} &= \left[ [1 - \alpha - G(\varphi_2^1)] + [\alpha + G(\varphi_2^1)] \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \right] \\ &\quad \cdot \left[ G(\varphi_1^1) T(\varphi_1^1) \left( \frac{1}{2} + \frac{\varphi_2^2 - \varphi_1^1 - \varphi_2^1}{2t_s} \right) - \frac{G(\varphi_1^1)}{2t_s} \right] \\ &\quad + \frac{\alpha}{4t_s} [G(\varphi_2^1) - G(\varphi_2^2)] \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) = 0.\end{aligned}\quad (A.5)$$

$$\begin{aligned}\frac{\partial F_2}{\partial \varphi_2^1} &= \left[ [1 - \alpha - G(\varphi_1^1)] + [\alpha + G(\varphi_1^1)] \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \\ &\quad \cdot \left[ G(\varphi_2^1) T(\varphi_2^1) \left( \frac{1}{2} + \frac{\varphi_2^2 - \varphi_1^1 - \varphi_2^1}{2t_s} \right) - \frac{G(\varphi_2^1)}{2t_s} \right] \\ &\quad + \frac{\alpha}{4t_s} G(\varphi_1^1) \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) \\ &\quad + \frac{G(\varphi_2^2)}{4t_s} \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] = 0.\end{aligned}\quad (A.6)$$

$$\begin{aligned}\frac{\partial F_2}{\partial \varphi_2^2} &= \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] \\ &\quad \cdot \left[ G(\varphi_2^2) T(\varphi_2^2) \left( \frac{1}{2} + \frac{\varphi_1^1 + \varphi_2^1 - \varphi_2^2}{2t_s} \right) - \frac{G(\varphi_2^2)}{2t_s} \right] \\ &\quad - \frac{[\alpha + G(\varphi_2^1)] G(\varphi_1^1)}{4t_s} \left( \frac{1}{2} + \frac{p_2 - p_1}{2t_p} \right) + \frac{G(\varphi_2^1)}{4t_s} \\ &\quad \cdot \left[ (1 - \alpha) + \alpha \left( \frac{1}{2} + \frac{p_1 - p_2}{2t_p} \right) \right] = 0.\end{aligned}\quad (A.7)$$

$$\begin{aligned}\frac{\partial F_1}{\partial p_1} &= X^1 D_1^1 + X^2 D_1^2 - \frac{(p_1 - c)}{2t_p} \{ X^1 [\alpha + G(\varphi_2^1)] [\alpha + G(\varphi_1^1)] \\ &\quad + X^2 [\alpha + G(\varphi_2^2)] \alpha \} = 0.\end{aligned}\quad (A.8)$$

$$\begin{aligned}\frac{\partial F_2}{\partial p_2} &= X^1 D_2^1 + X^2 D_2^2 - \frac{(p_2 - c)}{2t_p} \{ X^1 [\alpha + G(\varphi_2^1)] [\alpha + G(\varphi_1^1)] \\ &\quad + X^2 [\alpha + G(\varphi_2^2)] \alpha \} = 0.\end{aligned}\quad (A.9)$$

When the condition of this lemma holds (i.e.,  $\lim_{\varphi \rightarrow 0} T(\varphi)G(\varphi) = 0$ ) it is easy to show that  $\varphi_1^1 = 0$  and  $\varphi_2^1 = \varphi_2^2 = T^{-1}(1/(2t_s))$  solves the first-order conditions (A.5)–(A.9). The condition  $T'(\varphi) < 0$  guarantees that this solution is unique. Hence the AS regime reduces to the ES regime as claimed.  $\square$

### 8. Summary

In this paper, we proposed a new approach to reduce the costly consequences associated with uncertainty of demand. By making an effort that encourages some customers of one product to switch to another, the total uncertainty of demand (measured as the sum of modified standard deviations) is reduced. Consequently, the company is able to improve its profit and, at the same time, raise the service level it provides to customers. It was (pleasantly) surprising to find out that even a small value of  $\alpha$  (proportion of switching customers) results in an impressive profit increase.

This result may also shed new light on the centralization problem. It was demonstrated in the literature that using a centralized configuration by consolidating several decentralized points of sale into one is beneficial (see, for example, Eppen 1979). However, managers often are reluctant to adopt such a solution, fearing that they will lose some customers who will find the new location not as accessible as the location they used before it was consolidated. The results of this work suggest that companies can enjoy most of the benefits of centralization and still maintain a decentralized configuration by attempting to encourage some customers who used to buy in one location to approach another location. This can be done by distributing coupons or small gifts what will make one location more favorable to some clients what originally visited another location. At the same time, no sales will be lost due to location inaccessibility.

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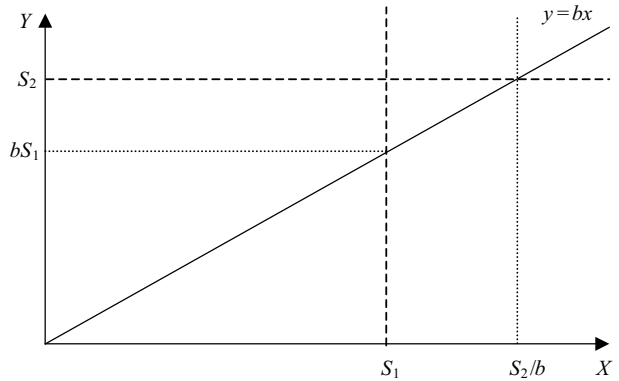
### Appendix

PROOF OF PROPERTY 1. AS the original demand varies over the domain  $X_0 \geq 0, Y_0 \geq 0$ , given a switching parameter,  $\alpha$ , the domain of the reshaped demand is  $X \geq 0, Y \geq bX$  (where  $b = \alpha/(1 - \alpha)$ ). Consequently, the calculations of  $\phi$  depend on the relationship between  $b$  and  $S_2/S_1$ .

Case I.  $b < S_2/S_1$ .

$$\phi = \int_{x=0}^{S_1} \int_{y=bx}^{bS_1} [p_1x + v_1(S_1 - x) + p_2y + v_2(S_2 - y)] \hat{f}(x, y) dy dx + \int_{x=0}^{S_1} \int_{y=bS_1}^{S_2} [p_1x + v_1(S_1 - x) + p_2y + v_2(S_2 - y)] \hat{f}(x, y) dy dx$$

Figure A1 The Regions of Integration (Case I)



$$\begin{aligned} &+ \int_{x=0}^{S_1} \int_{y=S_2}^{\infty} [p_1x + v_1(S_1 - x) + p_2S_2 - q_2(y - S_2)] \hat{f}(x, y) dy dx \\ &+ \int_{x=S_1}^{S_2/b} \int_{y=bx}^{S_2} [p_1S_1 - q_1(x - S_1) + p_2y + v_2(S_2 - y)] \hat{f}(x, y) dy dx \\ &+ \int_{x=S_1}^{S_2/b} \int_{y=S_2}^{\infty} [p_1S_1 - q_1(x - S_1) + p_2S_2 - q_2(y - S_2)] \hat{f}(x, y) dy dx \\ &+ \int_{x=S_2/b}^{\infty} \int_{y=bx}^{\infty} [p_1S_1 - q_1(x - S_1) + p_2S_2 - q_2(y - S_2)] \hat{f}(x, y) dy dx \\ &\quad - c_1S_1 - c_2S_2 \\ &= \int_{x=0}^{S_1} \int_{y=bx}^{bS_1} [p_1x + v_1(S_1 - x)] \hat{f}(x, y) dy dx \\ &\quad + \int_{x=0}^{S_1} \int_{y=bS_1}^{S_2} [p_1x + v_1(S_1 - x)] \hat{f}(x, y) dy dx \\ &\quad + \int_{x=0}^{S_1} \int_{y=S_2}^{\infty} [p_1x + v_1(S_1 - x)] \hat{f}(x, y) dy dx \\ &\quad + \int_{x=S_1}^{S_2/b} \int_{y=bx}^{S_2} [p_1S_1 - q_1(x - S_1)] \hat{f}(x, y) dy dx \\ &\quad + \int_{x=S_1}^{S_2/b} \int_{y=S_2}^{\infty} [p_1S_1 - q_1(x - S_1)] \hat{f}(x, y) dy dx \\ &\quad + \int_{x=S_2/b}^{\infty} \int_{y=bx}^{\infty} [p_1S_1 - q_1(x - S_1)] \hat{f}(x, y) dy dx - c_1S_1 \\ &\quad + \int_{y=0}^{bS_1} \int_{x=0}^{y/b} [p_2y + v_2(S_2 - y)] \hat{f}(x, y) dx dy \\ &\quad + \int_{y=bS_1}^{S_2} \int_{x=0}^{S_1} [p_2y + v_2(S_2 - y)] \hat{f}(x, y) dx dy \\ &\quad + \int_{y=S_2}^{\infty} \int_{x=0}^{S_1} [p_2S_2 - q_2(y - S_2)] \hat{f}(x, y) dx dy \\ &\quad + \int_{y=bS_1}^{S_2} \int_{x=S_1}^{y/b} [p_2y + v_2(S_2 - y)] \hat{f}(x, y) dx dy \\ &\quad + \int_{y=S_2}^{\infty} \int_{x=S_1}^{S_2/b} [p_2S_2 - q_2(y - S_2)] \hat{f}(x, y) dx dy \\ &\quad + \int_{y=S_2}^{\infty} \int_{x=S_2/b}^{y/b} [p_2S_2 - q_2(y - S_2)] \hat{f}(x, y) dx dy - c_2S_2 \end{aligned}$$

Futhermore,

$$\begin{aligned} E(U^2) &= \int_{u=0}^{\infty} u^2 f_U(u) du = \int_{u=0}^{\infty} u^2 \int_{x=0}^{\infty} g(u|x)h(x) dx du \\ &= \int_{x=0}^{\infty} \int_{u=0}^{\infty} u^2 g(u|x) du h(x) dx = \int_{x=0}^{\infty} E(U^2|x)h(x) dx. \end{aligned}$$

The second moment of any random variable with mean and standard deviation of  $\mu$  and  $\sigma$ , respectively, can be expressed as  $\sigma^2 + \mu^2$ . Furthermore, given  $x$  the variance of a binomial process is  $x\alpha(1-\alpha)$ . Therefore,

$$\begin{aligned} E(U^2) &= \int_{x=0}^{\infty} [x\alpha(1-\alpha) + (\alpha x)^2]h(x) dx \\ &= \alpha(1-\alpha) \int_{x=0}^{\infty} xh(x) dx + \alpha^2 \int_{x=0}^{\infty} x^2h(x) dx \\ &= \alpha(1-\alpha)\mu + \alpha^2(\sigma^2 + \mu^2). \end{aligned}$$

Finally,  $\text{Var}(U) = E(U^2) - E^2(U) = \alpha(1-\alpha)\mu + \alpha^2(\sigma^2 + \mu^2) - (\alpha\mu)^2 = \alpha(1-\alpha)\mu + \alpha^2\sigma^2$ .  $\square$

PROOF OF PROPERTY 6. In our process, we have two original random variables  $X$  and  $Y$ , which are the demands for Products 1 and 2, respectively. Clearly,  $X$  and  $Y$  may be correlated. Furthermore,  $X$  and  $Y$  may be defined as functions of three independent random variables  $A$ ,  $B$  and  $C$ , such that  $X = A + B$ , and  $Y = C + B$  ( $B$  is included in both  $X$  and  $Y$  to introduce correlation).

To maintain the parameters of  $X$  and  $Y$  (i.e.,  $\mu_1, \sigma_1, \mu_2, \sigma_2$ , and  $\rho$ ), it can easily be verified that  $\mu_1 = \mu_A + \mu_B, \mu_2 = \mu_B + \mu_C, \sigma_1^2 = \sigma_A^2 + \sigma_B^2$ , and  $\sigma_2^2 = \sigma_B^2 + \sigma_C^2$ . Furthermore,

$$\begin{aligned} \text{Cov}(X, Y) &= \text{Cov}(A + B, B + C) \\ &= \text{Cov}(A, B) + \text{Cov}(A, C) + \text{Cov}(B, C) + V(B) \\ &= 0 + 0 + 0 + V(B) = V(B). \end{aligned}$$

Hence,

$$\rho = \frac{\text{Cov}(X, Y)}{\sigma_1\sigma_2} = \frac{V(B)}{\sigma_1\sigma_2}$$

and  $\sigma_B^2 = V(B) = \rho\sigma_1\sigma_2$ .

The decision not to switch to another product may be viewed as a random selection with probability  $1-\alpha$ . Hence,  $\widehat{X}$  is the sum of two such random selections over  $A$  and  $B$ . Let  $A''$  and  $B''$  denote these random variables. Then, based on the lemma, it follows that  $\hat{\sigma}_1^2 = \sigma_{A''}^2 + \sigma_{B''}^2 = \alpha(1-\alpha)\mu_A + (1-\alpha)^2\sigma_A^2 + \alpha(1-\alpha)\mu_B + (1-\alpha)^2\sigma_B^2 = \alpha(1-\alpha)\mu_1 + (1-\alpha)^2\sigma_1^2$ .

$\widehat{Y}$ , on the other hand, is the sum of four random variables:  $A'$  (random selection with probability  $\alpha$  over  $A$ ),  $B'$  (random selection with probability  $\alpha$  over  $B$ ),  $B$ , and  $C$ . All of these variables, except for  $B'$  and  $B$ , are independent.

$$V(B'') = V(B - B') = V(B) + V(B') - 2\text{Cov}(B, B'),$$

or

$$\text{Cov}(B, B') = [V(B) + V(B') - V(B'')]/2.$$

Based on the lemma,

$$\sigma_{A'}^2 = \alpha(1-\alpha)\mu_A + \alpha^2\sigma_A^2, \quad \sigma_{B'}^2 = \alpha(1-\alpha)\mu_B + \alpha^2\sigma_B^2.$$

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Hence,

$$\begin{aligned} \text{Cov}(B, B') &= [\sigma_B^2 + \alpha(1-\alpha)\mu_B + \alpha^2\sigma_B^2 - \alpha(1-\alpha)\mu_B - (1-\alpha)^2\sigma_B^2]/2 \\ &= \alpha\sigma_B^2. \end{aligned}$$

Consequently,

$$\begin{aligned} V(B' + B) &= V(B') + V(B) + 2\text{Cov}(B, B') \\ &= \alpha(1-\alpha)\mu_B + \alpha^2\sigma_B^2 + \sigma_B^2 + 2\alpha\sigma_B^2 \\ &= \alpha(1-\alpha)\mu_B + (1+\alpha)^2\sigma_B^2, \end{aligned}$$

and finally,

$$\begin{aligned} \hat{\sigma}_2^2 &= V(\widehat{Y}) = V(A') + V(B' + B) + V(C) \\ &= \alpha(1-\alpha)\mu_A + \alpha^2\sigma_A^2 + \alpha(1-\alpha)\mu_B + (1+\alpha)^2\sigma_B^2 + \sigma_C^2 \\ &= \alpha(1-\alpha)\mu_1 + \alpha^2(\sigma_1^2 - \sigma_B^2) + (1+\alpha)^2\sigma_B^2 + \sigma_2^2 - \sigma_B^2 \\ &= \alpha(1-\alpha)\mu_1 + \alpha^2\sigma_1^2 + 2\alpha\sigma_B^2 + \sigma_2^2 \\ &= \alpha(1-\alpha)\mu_1 + \alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2. \quad \square \end{aligned}$$

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c.d.f. levels corresponds to a  $z$  value which can be used to express the optimal beginning inventory levels as  $S_1 = \hat{\mu}_1 + z_1 \hat{\sigma}_1$  and  $S_2 = \hat{\mu}_2 + z_2 \hat{\sigma}_2$ .

PROPERTY 2. The optimal  $z$  values are independent of the reshape factor,  $\alpha$ , and the correlation coefficient,  $\rho$ .

PROOF.  $\hat{F}_i(S_i)$  is a function of only the price/cost structure and not the demand distribution.  $\square$

Given the assumption of normally distributed demand, each of the two independent problems can be expressed as

$$\phi_i = (p_i - v_i)\hat{\mu}_i - (c_i - v_i)S_i - (p_i - v_i + q_i)\hat{\sigma}_i L(z_i).$$

See Silver et al. (1998, p. 388), where

$$L(z_i) = \int_{x=z_i}^{\infty} (x - z_i) f_s(x) dx$$

is the loss function, and

$$f_s(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

is the standard normal distribution. Using  $S_i = \hat{\mu}_i + z_i \hat{\sigma}_i$ , the profit can be rewritten as

$$\phi_i = (p_i - c_i)\hat{\mu}_i - [(c_i - v_i)z_i + (p_i - v_i + q_i)L(z_i)]\hat{\sigma}_i. \quad (5)$$

Using the expressions for  $\hat{\mu}_1, \hat{\mu}_2, \hat{\sigma}_1$ , and  $\hat{\sigma}_2$ , (1)–(4), the profit function can be expressed in terms of the original parameters as

$$\begin{aligned} \phi &= (p_1 - c_1)(1 - \alpha)\mu_1 - [(c_1 - v_1)z_1 + (p_1 - v_1 + q_1)L(z_1)] \\ &\quad \cdot (1 - \alpha)\sigma_1 + (p_2 - c_2)(\mu_2 + \alpha\mu_1) \\ &\quad - [(c_2 - v_2)z_2 + (p_2 - v_2 + q_2)L(z_2)] \\ &\quad \cdot \sqrt{\alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2}. \end{aligned}$$

The practice of demand reshape affects the profit in two ways: (1) reduction in the sum of demand variabilities, which reduces the need for safety stock inventory, and (2) by choosing one product versus another, there may be a change (increase or decrease) in the revenue (due to different selling prices, i.e.,  $p_1$  versus  $p_2$ ) or the cost (due to different unit cost, i.e.,  $c_1$  versus  $c_2$ ). To concentrate on the effect of demand variability we have neutralized the effect of price and

cost by using a uniform price/cost structure (i.e.,  $p_1 = p_2 = p, c_1 = c_2 = c, v_1 = v_2 = v, q_1 = q_2 = q$ ). Then, the profit function becomes

$$\phi = (p - c)(\mu_1 + \mu_2) - [(c - v)z + (p - v + q)L(z)]\sigma(\alpha), \quad (6)$$

where  $\sigma(\alpha) = (1 - \alpha)\sigma_1 + \sqrt{\alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2}$ . This expression can be used to demonstrate the shape and direction of the function with respect to the switching parameter,  $\alpha$ , as suggested in the following property.

PROPERTY 3.  $\phi$  is increasing in  $\alpha$ , and concave.

PROOF.

$$\begin{aligned} L(z) &= \int_{x=z}^{\infty} (x - z) f_s(x) dx \\ &= \int_{x=z}^{\infty} x f_s(x) dx - z \int_{x=z}^{\infty} f_s(x) dx \\ &= \int_{x=z}^{\infty} x f_s(x) dx - z[1 - F(z)], \end{aligned}$$

where  $F(z) = \int_{x=-\infty}^z f_s(x) dx$ . Because at optimality

$$\hat{F}_i(S_i) = \frac{p - c + q}{p - v + q} (= F(z)),$$

it follows that at optimality

$$(c - v)z + (p - v + q)L(z) = (p - v + q) \int_{x=z}^{\infty} x f_s(x) dx > 0.$$

Therefore,

$$\begin{aligned} \frac{d\phi}{d\alpha} &= -[(c - v)z + (p - v + q)L(z)] \\ &\quad \cdot \left[ -\sigma_1 + \frac{\alpha\sigma_1^2 + \rho\sigma_1\sigma_2}{\sqrt{\alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2}} \right] \\ &= [(c - v)z + (p - v + q)L(z)]\sigma_1 \\ &\quad \cdot \left[ 1 - \frac{\alpha\sigma_1 + \rho\sigma_2}{\sqrt{\alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2}} \right] > 0, \end{aligned}$$

$$\begin{aligned} \frac{d^2\phi}{d\alpha^2} &= -[(c - v)z + (p - v + q)L(z)]\sigma_1 \\ &\quad \cdot \left[ \left( \sigma_1 \sqrt{\alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2} - (\alpha\sigma_1 + \rho\sigma_2) \right) \right. \\ &\quad \cdot \left( \frac{\alpha\sigma_1^2 + \rho\sigma_1\sigma_2}{\left( \sqrt{\alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2} \right)} \right) \\ &\quad \left. / \left( \alpha^2\sigma_1^2 + 2\alpha\rho\sigma_1\sigma_2 + \sigma_2^2 \right) \right] \\ &= -[(c - v)z + (p - v + q)L(z)]\sigma_1 \end{aligned}$$

N3. The three-leg network described in the example of §4.1, with four nodes: a hub  $h$ , two origin nodes  $o_1, o_2$ , and a destination node  $d$ , and the legs (1)  $o_1h$ , (2)  $o_2h$ , (3)  $hd$ . There is demand from the origin nodes  $o_1, o_2$  to the hub node  $h$  and to the destination node  $d$ , on the itineraries: (1)  $o_1h$ , (2)  $o_2h$ , (13)  $o_1hd$ , (23)  $o_2hd$ . Suppose that there is only one fare class per itinerary, and there is no demand from  $h$  to  $d$ . The leg-class incidence matrix, together with the fare structure  $\mathbf{R}$ , is:

$$\begin{pmatrix} \mathbf{R} \\ \mathbf{A} \end{pmatrix} = \begin{pmatrix} R_1 & R_2 & R_{13} & R_{23} \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

N4. A four-leg network, with two origins  $o_1, o_2$ , 2 destinations  $d_1, d_2$  and a hub  $h$ . The legs in the network are (1)  $o_1h$ , (2)  $o_2h$ , (3)  $hd_1$ , (4)  $hd_2$ , with capacities  $\mathbf{N} = (N_1, N_2, N_3, N_4)$ . There is demand for all the eight itineraries, with one fare class per itinerary. The leg-class incidence matrix, together with the fare structure  $\mathbf{R}$ , is:

$$\begin{pmatrix} \mathbf{R} \\ \mathbf{A} \end{pmatrix} = \begin{pmatrix} R_1 & R_2 & R_3 & R_4 & R_{13} & R_{14} & R_{23} & R_{24} \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

N4.2. The same as (N4), except there are two fares per itinerary, so 16 demand classes in all. The leg-class incidence matrix, together with a high-low fare structure  $\mathbf{R} = (\mathbf{R}^h, \mathbf{R}^l)$ , is:

$$\begin{pmatrix} \mathbf{R} \\ \mathbf{A} \end{pmatrix} = \begin{pmatrix} R_1^l & R_1^h & R_2^l & R_2^h & R_3^l & R_3^h & R_4^l & R_4^h & R_{13}^l & R_{13}^h & R_{14}^l & R_{14}^h & R_{23}^l & R_{23}^h & R_{24}^l & R_{24}^h \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

We used the following alternative scenarios to model the arrival process:

*HP.* Homogeneous Poisson arrivals with arrival rates given by a constant vector  $\mathbf{p}$  (i.i.d. Bernoulli trials).

*NHL.* Nonhomogeneous Poisson with hi-low demand. We assume arrival rates increase for high-fare classes ( $\mathbf{p}^t = \mathbf{p}/\log_a(a + t \cdot \rho)$ ) and decrease ( $\mathbf{p}^t = \mathbf{p}/\log_a(a + (T - t) \cdot \rho)$ ) for low-fare classes, as we approach departure.

*NLH.* Nonhomogeneous Poisson with low-hi demand. We assume arrival rates decrease for high-fare classes ( $\mathbf{p}^t = \mathbf{p}/\log_a(a + (T - t) \cdot \rho)$ ) and increase ( $\mathbf{p}^t = \mathbf{p}/\log_a(a + t \cdot \rho)$ ) for low-fare classes, as we approach departure.

The log-factors are given by  $\rho$ , and  $a$  is a constant, equal to the base of the logarithm (we take  $a = 2$ ).

**6.1.2. Running Time.** Exact calculations of the optimal expected revenue (DP), and expected values of the proposed policies (CEC, BPC) are practically impossible. Already for two-leg networks (N2) with one class per itinerary, 50 seats per leg initial capacity and  $T = 300$  time periods the computation takes about two hours.

A tractable approach for measuring performance of the proposed policies, however, is provided by simulation. A simulation run of the CEC or BPC policies for a two-leg network takes less than a minute (less than a second per iteration). The largest network we simulated was a four-leg network (N4.2) with two demand classes per itinerary. When the initial capacities are in the order of  $N = 50 - 100$  and the time horizon has  $T = 300$  time periods, a full simulation run for such an instance takes a few minutes (i.e., a few seconds per iteration).